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PERSISTENCE OF EXTREMELY WET AND EXTREMELY DRY MONTHS IN THE UNITED STATES [1]

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ABSTRACT

Statistical tests based on comparisons of observed occurrences of extremely wet and extremely dry months at 46 stations and 4 areas in the United States with occurrences predicted by the binomial theorem indicate that there is a tendency for persistency of extremely dry months.

INTRODUCTION

Many of the statistical problems in climatology involve the question of dependence or independence between precipitation amounts for consecutive periods. This paper presents a study of consecutive monthly precipitation amounts, which will follow a review of results obtained by other investigators for similar studies.

REVIEW OF EARLIER RESULTS

Gold [1] compared the frequencies of isolated sequences of "rain" and "fine" days at Kew, England, over a 10-year period with the probable number of sequences of 1, 2, 3, etc., days that pure chance would give. His conclusions were that frequencies of sequences of 6, 7, 8, 9, and 10 days occurred more often than pure chance would indicate. Hawke [2] applied Gold's probability numbers to "wet" and "dry" months from 1841 to 1930 at Greenwich. He showed that sequences of 1, 3, 5, 6, and 15 months of the same precipitation character occurred more often than expected but not significantly so. Bilham [3] also used Gold's chance probabilities for wet (above normal) and dry (below normal) months over England and Wales. His conclusions were: 1. Isolated dry and wet months were less frequent than chance would indicate. 2. Runs

of five or more months (wet or dry) occurred more frequently than indicated by chance. 3. Runs of three or more dry months occurred distinctly more frequently than runs of three or more wet months.

Cochran [4] extended Gold's probabilities so that they would include cases where the chance occurrences of two events are not equal. He set up table 1 in which wet and dry months are defined as being above and below normal, respectively. Expected frequencies, computed from marginal totals, are shown in parentheses.

TABLE 1.—*Contingency table showing frequency of wet and dry current months against wet and dry preceding months. Expected frequencies are shown in parentheses. England and Wales, 1727–1934 (after Cochran)*

| | | Preceding month | | Total |
		Wet	Dry	
Current month	Wet	542 (513)	582 (611)	1,124
	Dry	582 (611)	759 (730)	1,341
	Total	1,124	1,341	2,465

From the table, the proportion of wet months if the previous month was wet is $542 \div 1124 = 0.4822$ with standard error $= \pm 0.01485$; the proportion of wet months if the previous month was dry is $582 \div 1341 = 0.4340$ with standard error $= \pm 0.01360$. Thus a tendency toward persistence is indicated. Cochran then found a standard

error of $\pm \sqrt{(0.01485)^2+(0.01360)^2}= \pm 0.02013$ for the difference between the fractions and points out that the probability that this or a greater difference should arise by chance is about 1 in 60. To make this result comparable with the method to be used in this paper, we computed from his data a χ^2-value of 5.55, which is significant at the 2 percent level for one degree of freedom[2]. Beer, Drummond, and Fürth [5], using the same British data, divided monthly, homogenous, long rainfall records at seven localities into two groups depending upon whether the month was above (wet) or below normal (dry) and obtained table 2.

TABLE 2.—*Contingency table showing frequency of wet and dry current months against wet and dry preceding months. Expected frequencies are shown in parentheses. British Isles 1815–1944 (after Beer, Drummond, and Fürth)*

| | | Preceding month | | Total |
		Wet	Dry	
Current month	Wet	200 (207)	266 (259)	466
	Dry	269 (262)	322 (329)	591
	Total	469	588	1,057

A χ^2-value of 0.588 with one degree of freedom indicates no significance at the 10 percent level.

Tannehill [6] noted that Washington, D. C., records for 1930 showed four consecutive months with precipitation amounts less than one inch: August, 0.62; September, 0.76; October, 0.28; and November, 0.79 in. Using 77 years of record he found only 4 Augusts, 6 Septembers, 12 Octobers, and 8 Novembers with less than one inch of precipitation. Under the assumption of independence, month for month, of the occurrence or nonoccurrence of monthly amounts less than 1 inch, this gave a chance value of about one in 15,000 of having each of these 4 months with less than 1 inch in the same year. He proposed that some persistent control modified the climate temporarily during the 1930 drought period.

Solot [7], investigating 40 years of Hawaiian Islands records, has presented the data in table 3 to show the lag relationship of rainfall of a given class for a given month with that of subsequent months. In this table, monthly rainfall amounts were divided into the three classes of equal observed frequency—heavy, moderate, and light. The middle class (not shown here) does not differ significantly from chance distribution, while light rainfall is significantly related to the occurrence of light rainfall up to approximately 3 months later and heavy rainfall is significantly related to subsequent heavy rainfall up to 2 months later.

[2] Many textbooks in statistics discuss the χ^2-test and its significance. For example, see P. G. Hoel, *Introduction to Mathematical Statistics*, John Wiley and Sons, New York, 1947, pp. 186–195. In judging the results of this and later applications of the χ^2-test, it should be remembered that the test is insensitive to type II error, i. e., the test may fail to show dependence when some degree of dependence is present. Many other statistical tests, of course, show a similar insensitivity; although with some types of data, it is possible to use tests that are somewhat more sensitive.

TABLE 3.—*Frequency of occurrence of rainfall classes in subsequent months in Hawaiian Islands, 1905–45 (after Solot)*

[Intervening months are not considered]

| Months later | Light followed by— | | | Heavy followed by— | | |
	Light	Moderate	Heavy	Light	Moderate	Heavy
	Percent	*Percent*	*Percent*	*Percent*	*Percent*	*Percent*
1	45	34	21	20	31	49
2	44	32	24	25	33	42
3	41	34	25	30	31	39
4	36	32	32	35	31	35
5	33	35	32	31	31	38

Lastly, Showalter [8] classified monthly and annual precipitation at Los Angeles into three categories (wet, normal, and dry) with equal chance of occurrence and compared the observed number of successive months or years of each class with the number expected to occur by chance. His results show that successive years and successive months of the same rain character have not occurred as frequently as would be expected by chance.

Summarizing: In England there seems to be a tendency toward persistence for wet and dry months, the tendency being slightly stronger for dry months. In Hawaii there is a definite tendency for both heavy and light monthly amounts to persist up to 2 or 3 months. The investigation of the Los Angeles area indicates no persistence for monthly rainfall amounts, while for the United States in general there have not been enough tests to permit definite conclusions.

STUDIES IN COLORADO RIVER BASIN

In a study sponsored by the United States Weather Bureau and the Bureau of Reclamation [9], from which this paper is an outgrowth, various tests of independence were used. The records of eight stations in the Colorado River Basin were checked and the frequency of occurrence of wet (above normal) and dry (below normal) October–December and January–April periods were tabulated as in table 4.

TABLE 4.—*Contingency table showing frequency of occurrence of wet and dry October–December and following January–April periods in Colorado River Basin, 1914–49. Expected frequencies are shown in parentheses*

| | | October–December | | Total |
		Wet	Dry	
January–April	Wet	66 (60)	68 (74)	134
	Dry	58 (64)	85 (79)	143
	Total	124	153	277

A χ^2-test on departures of actual from expected frequencies (in parentheses) gave a value of 2.2 which is not significant at the 5 percent level. A similar test on October–December, January–February, and March–April precipitation frequencies yielded a χ^2-value which indicated no dependence at the 1 percent level. Correlation tests

TABLE 5.—*Correlations for average precipitation for certain periods vs. following indicated periods, Eastern Utah, 1892–1947*

Periods	Correlation coefficient
October–December vs. January–April	0.27
October–December vs. January–February	.03
October–December vs. March–April	.31
January–February vs. March–April	.10
October vs. November	.07
November vs. December	.18
December vs. January	.03
January vs. February	.16
February vs. March	.03
March vs. April	−.16

were then made for 56 years of average precipitation over eastern Utah (table 5).

For 56 items at the 5 and 1 percent levels the lowest significant coefficients are 0.259 and 0.377, respectively. The z-test applied to these coefficients individually would indicate that they are not significant. The fact that 9 of the 10 are positive, though, is suggestive of some tendency toward persistence.

EXTREMELY WET AND EXTREMELY DRY MONTHS

For a more accurate estimate of dependence of extreme values it was decided to concentrate on a study of highest and lowest precipitation amounts. Using the 50 years (1898–1947) of rainfall records at various stations in the United States, the five highest and five lowest monthly totals of precipitation for each month at each station were designated as A's and C's, respectively; all others were designated as B's. These data were prepared for a total of 46 stations and 4 areas (table 6). Of course, the data are not to be regarded as representing 50 independent stations since precipitation amounts at some of the stations are undoubtedly correlated. An example of the data for one station is shown in table 7. Stations in areas such as California were eliminated because the absence of rainfall in most summers makes it impossible to determine the five extremely low months. If a choice was necessary in designating the fifth highest or lowest value, it was standard procedure to choose the one of earliest date. With the extremely wet and extremely

TABLE 6.—*Stations and areas for which extremely high and extremely low monthly precipitation amounts were determined*

1. Albany, N. Y.	26. New York, N. Y.
2. Alpena, Mich.	27. North Head, Wash.
3. Atlanta, Ga.	28. North Platte, Nebr.
4. Baker, Oreg.	29. Oklahoma City, Okla.
5. Bismarck, N. Dak.	30. Omaha, Nebr.
6. Boston, Mass.	31. Philadelphia, Pa.
7. Cheyenne, Wyo.	32. Portland, Oreg.
8. Chicago, Ill.	33. Pueblo, Colo.
9. Cincinnati, Ohio.	34. Rapid City, S. Dak.
10. Columbia, Mo.	35. St. Louis, Mo.
11. Des Moines, Iowa.	36. Salt Lake City, Utah.
12. Detroit, Mich.	37. Sault Ste. Marie, Mich.
13. Dodge City, Kans.	38. Seattle, Wash.
14. Grand Rapids, Mich.	39. Sioux City, Iowa.
15. Green Bay, Wis.	40. Spokane, Wash.
16. Helena, Mont.	41. Tacoma, Wash.
17. Huron, S. Dak.	42. Topeka, Kans.
18. Kansas City, Mo.	43. Tulsa, Okla.
19. Keokuk, Iowa.	44. Valentine, Nebr.
20. Lander, Wyo.	45. Washington, D. C.
21. Little Rock, Ark.	46. Wichita, Kans.
22. Madison, Wis.	47. Eastern Oklahoma.
23. Milwaukee, Wis.	48. Central Oklahoma.
24. Minneapolis, Minn.	49. Western Oklahoma.
25. New Orleans, La.	50. Eastern Utah.

TABLE 7.—*Calendar of extremely wet and extremely dry months at Pueblo, Colo. For each month, A's designate 5 highest monthly precipitation totals, C's the 5 lowest, and B's all others*

Year	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
1898	B	B	B	B	A	B	C	B	B	B	B	A
99	B	B	B	C	C	B	A	B	B	B	B	B
1900	B	B	B	A	B	B	B	C	C	B	B	B
01	B	B	B	B	A	B	B	B	B	B	C	B
02	B	B	B	C	B	B	B	B	B	B	B	B
03	B	A	B	B	C	A	B	B	B	B	B	B
04	B	B	B	B	A	A	B	B	B	C	C	B
05	B	B	A	A	B	B	B	B	B	B	B	B
06	B	C	B	B	B	B	B	B	B	B	B	C
07	B	B	C	B	B	B	B	B	B	B	B	B
08	B	B	B	C	B	C	B	B	B	B	B	C
09	B	B	B	B	B	B	B	B	A	B	A	B
1910	B	B	C	B	B	B	B	B	B	C	B	B
11	B	B	B	B	B	B	B	C	B	B	B	A
12	B	B	B	B	B	B	B	B	B	B	C	C
13	B	A	B	B	B	B	B	B	B	B	B	A
14	B	B	B	A	A	B	A	B	B	A	C	B
15	B	B	A	A	B	B	B	B	A	B	B	B
16	B	C	B	B	B	B	B	B	C	B	B	B
17	B	B	B	B	B	B	B	B	A	B	C	B
18	B	B	B	B	C	B	B	B	A	B	B	B
19	C	B	A	B	C	B	B	B	A	B	B	B
1920	B	B	C	B	B	B	B	B	B	B	B	B
21	B	B	B	B	A	A	B	B	B	B	A	A
22	B	B	B	B	B	B	C	B	C	B	A	C
23	C	B	B	B	B	A	A	B	A	B	B	B
24	A	B	B	B	B	B	B	C	C	B	B	B
25	A	B	B	C	B	B	A	B	B	B	A	B
26	A	C	B	B	B	B	B	B	B	B	B	B
27	B	B	B	C	B	B	B	B	C	B	B	B
28	C	B	A	B	A	B	B	A	B	A	B	B
29	B	B	B	B	B	C	B	A	B	B	B	B
1930	B	C	B	B	B	B	B	B	B	B	B	B
31	B	A	B	B	B	B	B	B	B	B	B	C
32	B	C	B	B	C	B	B	B	B	C	B	B
33	C	B	B	B	B	B	B	B	B	C	B	B
34	C	A	B	B	B	C	B	B	B	C	B	B
35	B	B	C	B	B	B	C	B	A	B	B	B
36	B	B	C	B	A	B	C	B	B	B	B	A
37	B	B	B	B	B	B	C	B	B	B	B	B
38	C	B	B	B	B	B	B	B	B	C	B	B
39	A	A	B	B	B	C	C	B	B	C	B	B
1940	A	B	B	B	B	B	B	B	A	B	B	B
41	B	B	A	B	B	B	B	A	B	A	B	B
42	B	B	B	A	B	A	B	B	B	A	B	B
43	B	B	B	B	B	B	B	B	B	B	B	B
44	B	B	A	B	B	B	B	B	B	B	B	B
45	A	B	B	B	B	B	B	A	B	B	B	B
46	B	B	B	B	B	C	B	A	B	B	A	B
47	B	B	B	B	B	A	B	B	B	A	B	B

dry months (A's and C's) thus determined, chance frequencies based on the binomial distribution were computed and various tests were made as to whether the A's and C's occur differently than chance would indicate for a like number of A's and C's under similar constraints, i. e., 5 A's and 5 C's in each of 12 vertical columns of 50 entries each.

FREQUENCY OF YEARS WITH FOUR OR MORE A's OR C's

The occurrence of a comparatively large number of A's or C's in some years with very few in others would be indicative of persistence, so one test consisted of counting the number of years with 4 or more A's or 4 or more C's at a station. Table 8 compares the observed and chance frequencies of the number of stations with 0, 1, 2, and 3 or more years with 4 or more A's and C's.

A χ^2-value of 1.603 on the A's is not significant at the 10 percent level for 3 degrees of freedom. On the C's, $\chi^2 = 3.574$ is also not significant at the 10 percent level. The fact that neither value is significant indicates that there

TABLE 8.—*Comparison of observed and chance number of stations having various numbers of years with 4 or more extremely wet or extremely dry months*

Years	Number of stations having indicated number of years with—		Number expected by chance
	4 or more A's	4 or more C's	
0	16	8	13.6
1	19	19	18.0
2	8	15	11.6
3 or more	7	8	6.8
Total	50	50	50.0

is no pronounced tendency for extremely wet or extremely dry months to persist during a year, that is, for extremely wet months or for extremely dry months to occur together in the same year.

OCCURRENCE OF EXCEPTIONALLY WET MONTHS AND EXCEPTIONALLY DRY MONTHS IN THE SAME YEAR

Another test of randomness is based on the distribution of various numbers of C's in the years in which certain numbers of A's have occurred, or, as we have tested, the frequencies of years with various numbers of C's in the same year with 2, 3, or 4 A's. Table 9 compares the observed and theoretical frequencies of occurrence of various numbers of C's and A's in years with exactly two A's or C's respectively, in the 50 years at the 46 stations and 4 areas.

TABLE 9.—*Comparison of observed and theoretical frequencies of C's and A's in years that have exactly 2 A's or 2 C's, respectively*

Number of C's in a year with 2 A's	Observed frequencies	Chance frequencies	Number of A's in a year with 2 C's	Observed frequencies	Chance frequencies
0	199	209	0	194	190
1	219	232	1	189	211
2	126	116	2	125	105
3	47	34	3	30	31
4 or more	8	8	4 or more	6	7
	599	599		544	544

With 4 degrees of freedom $\chi^2=2.566$, which is not significant at the 10 percent level. | With 4 degrees of freedom $\chi^2=6.338$, which is not significant at the 10 percent level.

Secondly, we compare the observed frequencies of C's or A's when 3 A's or 3 C's, respectively, have occurred with the theoretical frequencies in Table 10.

TABLE 10.—*Comparison of observed and theoretical frequencies of C's and A's in years that have exactly 3 A's or 3 C's, respectively*

Number of C's in a year with 3 A's	Observed frequencies	Chance frequencies	Number of A's in a year with 3 C's	Observed frequencies	Chance frequencies
0	92	85	0	88	86
1	84	85	1	72	86
2	30	38	2	49	38
3 or more	14	12	3 or more	13	12
	220	220		222	222

$\chi^2=2.290$, which with 3 degrees of freedom is not significant at the 10 percent level. | $\chi^2=3.734$, which with 3 degrees of freedom is not significant at the 10 percent level.

Lastly, in table 11 we compare the observed frequencies of years with C's or A's when 4 A's or 4 C's, respectively, have occurred with the theoretical frequencies of such occurrences.

TABLE 11.—*Comparison of observed and theoretical frequencies of C's and A's in years that have exactly 4 A's or 4 C's, respectively*

Number of C's in years with 4 A's	Observed frequencies	Chance frequencies	Number of A's in years with 4 C's	Observed frequencies	Chance frequencies
0	22	21	0	28	25
1	21	20	1	18	22
2 or more	8	10	2 or more	12	11
	51	51		58	58

$\chi^2=0.498$, which with 2 degrees of freedom is not significant at the 10 percent level. | $\chi^2=1.178$, which with 2 degrees of freedom is not significant at the 10 percent level.

These tests on randomness of C's and A's in a year indicate no tendency for them to occur with a frequency very much different from that which pure chance would allow.

DISTRIBUTION OF PAIRS

The above comparisons test the tendency for persistence on a yearly basis; they would not very well indicate persistence of shorter periods. Therefore, probabilities of numbers of pairs were devised to give the chance probabilities of having two exceptionally wet months consecutively (A in one month followed by an A in the next) or of having two exceptionally dry months in succession (a C followed by a C). A frequency table of the number of pairs of extremely wet and extremely dry amounts at the 46 stations and 4 areas appears below with the corresponding chance frequencies. Because a χ^2-test is considered to be less reliable if cell frequencies are less than 5, several numbers of pairs are grouped.

TABLE 12.—*Comparison of observed and theoretical frequencies of stations having various numbers of pairs of A's and C's*

Pairs of A's	Observed number of stations	Chance frequencies	Pairs of C's	Observed number of stations	Chance frequencies
0–3	7	6.4	0–3	5	6.4
4–5	17	15.3	4–5	8	15.3
6	8	8.8	6	6	8.8
7	7	7.6	7	7	7.6
8 or more	11	11.9	8	6	5.4
			9 or more	18	6.5
	50	50.0		50	50.0

This gave a value of 0.434 for χ^2, which for 4 degrees of freedom is almost significant at the 99 percent level, indicating very close agreement with expected frequencies. | Here $\chi^2=25.09$ and with 5 degrees of freedom is significant at the 0.1 percent level.

Mann and Wald [10] have recommended that for a more reliable χ^2-test the theoretical frequencies of each class be as nearly equal as possible. Accordingly, the data in table 12 were regrouped as shown in table 13.

From the tests on the distribution of pairs of extremely wet and extremely dry precipitation amounts it is indicated that:

TABLE 13.—*Comparison of observed and theoretical frequencies of stations having various numbers of pairs of A's and C's*

Pairs of A's	Observed number of stations	Chance frequencies	Pairs of C's	Observed number of stations	Chance frequencies
0–5	24	21.7	0–5	13	21.7
6 or more	26	28.3	6 or more	37	28.3
This resulted in 0.431 for χ^2, which with 1 degree of freedom falls near the 50 percent level.			χ^2 is now 6.17, which is almost significant at the 1 percent level with 1 degree of freedom.		

(1) Pairs of extremely wet amounts occur very much as would be expected by chance.

(2) Pairs of extremely dry amounts occur more often than chance frequency.

CONCLUSIONS

1. Various tests indicate that in the United States the occurrence of pairs of extremely wet months is not significantly different from chance so far as the χ^2-test can determine; however, the same tests indicate that pairs of extremely dry months occur more often than chance would predict.

2. The tests used indicate that the distribution of extremely wet and extremely dry months by years is not significantly different from chance in the United States—again subject to the limitations of the χ^2-test.

REFERENCES

1. E. Gold, "Note on the Frequency of Occurrence of Sequences in a Series of Events of Two Types," *Quarterly Journal of the Royal Meteorological Society*, vol. 55, No. 231, July 1929, pp. 307–309.

2. E. L. Hawke, "Frequency Distribution of Wet and Dry Months from 1815 to 1914 and of Warm and Cold Months for 1841–1930 at Greenwich," *Quarterly Journal of the Royal Meteorological Society*, vol. 60, No. 253, January 1934, pp. 71–73.

3. E. G. Bilham, "Notes on Sequences of Dry and Wet Months in England and Wales," *Quarterly Journal of the Royal Meteorological Society*, vol. 60, No. 257, October 1934, pp. 514–516.

"Sequences of Dry and Wet Months in England and Wales," *Quarterly Journal of the Royal Meteorological Society*, vol. 64, No. 275, April 1938, p. 324.

4. W. G. Cochran, "An Extension of Gold's Method of Examining the Apparent Persistence of One Type of Weather," *Quarterly Journal of the Royal Meteorological Society*, vol. 64, No. 277, October 1938, pp. 631–634.

5. A. Beer, A. Drummond, and R. Fürth, "Sequences of Wet and Dry Months and the Theory of Probability," *Quarterly Journal of the Royal Meteorological Society*, vol. 72, No. 311, January 1946, pp. 74–86.

6. I. R. Tannehill, *Drought; Its Causes and Effects*, Princeton University Press, Princeton 1947, pp. 40–41.

7. S. B. Solot, "Possibility of Long Range Precipitation Forecasting for the Hawaiian Islands," *Research Paper No. 28*, U. S. Weather Bureau, Washington, 1948.

8. A. K. Showalter, "Chance for Successive Dry Years in Southern California," *Monthly Weather Review*, vol. 76, No. 10, October 1948, pp. 221–223.

9. U. S. Weather Bureau and Bureau of Reclamation, "Maximum Possible Flood-Producing Meteorological Conditions, Colorado River Basin," *Cooperative Studies Report No. 9*, Washington, 1949.

10. H. B. Mann and A. Wald, "On the Choice of the Number of Class Intervals in the Application of the Chi-Square Test," *Annals of Mathematical Statistics*, vol. 13, No. 3, September 1942, pp. 306–317.